Electromagnetic diffraction of light focused through a planar interface between materials of mismatched refractive indices: an integral representation

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The diffraction of electromagnetic waves for light focused by a high numerical aperture lens from a first material into a second material is treated. The second material has a different refractive index from that of the first material and introduces spherical aberration. We solve the diffraction problem for the case of a planar interface between two isotropic and homogeneous materials with this interface perpendicular to the optical axis. The solution is obtained in a rigorous mathematical manner, and it satisfies the homogeneous wave equation. The electric and magnetic strength vectors are determined in the second material. The solution is in a simple form that can be readily used for numerical computation. A physical interpretation of the results is given, and the paraxial approximation of the solution is derived.

1. INTRODUCTION

The structure of focused electromagnetic waves has been studied by a number of authors. Of particular interest from both the theoretical and practical points of view is the diffracted-light distribution in the region of focus when light is focused by a glass lens. The results of such investigations have helped in the design of better objective lenses, especially those with high numerical apertures.

Focusing through materials of mismatched refractive indices has attracted particular interest because of microscopy applications in the biological and material sciences. Microscope objective lenses have long been manufactured with aberration correction for certain but fixed penetration depths, e.g., lenses to be used with a cover glass for biological applications. Microscope objective lenses have recently been produced that provide continuously variable correction for spherical aberration introduced by off-axis propagation of light through materials of different refractive indices. Nevertheless, the importance of investigations directed at a better understanding of high aperture focusing is still great, and this demand is increasing, especially with new material science applications.

The literature that deals with the general focusing problems of electromagnetic waves is well established. In an early paper Wolf[1] treated high aperture focusing of electromagnetic waves in a single homogeneous material. The starting point of his elegant theory was the representation of the angular spectrum of plane waves, from which integral formulas similar to the Debye integral[2] were obtained. Wolf and Li[3] subsequently showed that this approach was valid for systems that satisfied the high-aperture condition. Although, during the derivation of his formulas, Wolf[1] made a number of approximations, his transformed integral formulas were formally identical to Luneburg's[4] representation of the Debye integral, which Luneburg had previously shown to be an exact solution of the homogeneous wave equation. Luneburg's formulation is valid for an idealized problem, i.e., when the probe function and the field to be determined have the same boundary values at infinity as their geometrical optics solution. Kant[5] recently showed how Wolf's integral formulas could be used when (Seidel) lens aberrations were present in the optical system.

The first paper that dealt with the focusing of electromagnetic waves into mismatched refractive index materials was by Gasper *et al.*,[6] who considered an arbitrary electromagnetic wave as it traversed a planar interface. They also used the representation of the angular spectrum of plane waves and considered the focusing problem for isotropic homogeneous media. For a treatment of diffraction in anisotropic materials the reader is referred to Stamnes and Sherman.[7,8] Gasper *et al.*[6] derived a rigorous solution of the problem and its asymptotic approximation and gave expressions for the electric and magnetic fields. However, because of the complexity of their theory, the use of their formulas for the computation of the electromagnetic field near focus was not practical in most cases. Ling and Lee[9] treated the focusing of electro-

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magnetic waves through an interface. A boundary condition in the form of a current distribution was used as the starting point, and the representation of the angular spectrum of plane waves was applied. In a semigeometrical approach, with the use of the stationary-phase method, expressions were obtained for the electric and magnetic fields. However, Ling and Lee used approximations to obtain their integral formulas. Ji and Hongo[10] treated the different problem of a point source and a spherical dielectric interface, using Maslov's method to obtain the electric field in the focal region. Although the final equations showed the same general form as those of the present paper, they could not be applied to the present problem. A comprehensive treatment of different focusing theories was later given by Stamnes.[11]

In a recent paper Hell *et al.*[12] considered the focusing problem for mismatched refractive index materials using the Fresnel–Kirchhoff integral. They decomposed the incident electric vector into *s*- and *p*-polarized parts and also calculated the effect of spherical aberration on the image formation for a confocal fluorescence microscope. However, the integral formula that they used is derived from Green's theorem, which requires the continuity of the field and its first and second derivatives within and on the boundary of the area of integration; this is not the case for the normal component of the electric field and the tangential component of the magnetic field, and so the final integral formulas obtained may not be rigorously correct.

The purpose of this paper is to extend Wolf's treatment of the diffraction problem for the case when an electromagnetic wave is focused in a single medium of propagation to the case when light is focused by a high-aperture system into a bulk specimen that has a refractive index different from that of the medium of propagation and introduces a considerable amount of spherical aberration. The starting point of our treatment is Wolf's integral formulas, with which we derive the electromagnetic field just before the interface between the two media. The field is then traversed across the interface by application of the Fresnel refraction law to the individual plane waves incident upon the interface. The field so derived just after the interface is used as the boundary condition for a second set of integral formulas corresponding to a superposition of plane waves, which represent the field inside the second medium. In this way the diffraction problem is solved in a rigorous mathematical manner, and the solution satisfies the homogeneous wave equation. We also obtain formulas for the strength vectors in the second material in a form more generally applicable than that of those previously published.

In our calculations the spherical aberration is introduced by the specimen rather than by an aberration function related to the lens. The reason is that modern high-performance lenses are generally well corrected for spherical aberration. Therefore, in considering a real optical system, we feel justified in the assumption of a perfect spherical wave emerging from the lens and converging toward the Gaussian focus point (with no spherical aberration). Although strong spherical aberrations can be treated by geometrical optics successfully describing the main features of the focal region (e.g., caustic and circle of least confusion), the fine structure and the electromagnetic properties (e.g., polarization and energy flow) of the field can be described only by a full electromagnetic treatment. This becomes increasingly important when the field so calculated is to be applied to wave scattering by small objects situated close to the focus and whose size is comparable with that of the wavelength, e.g., in biological and material science applications.

The presentation of our paper is as follows: In Subsection 2.A we derive the integral representation of the electromagnetic field in the image space. In Subsection 2.B we decompose the electric and magnetic vectors as each traverses the optical system. In Subsection 2.C we make use of the above results and give simplified formulas for the electric and magnetic fields in the second material. Finally, in Subsections 2.D and 2.E we consider the physical interpretation of our formulas and give a paraxial approximation of our solution.

2. PROBLEM, FORMULATION, AND SOLUTION

A. Integral Representation

Consider an optical system of revolution with an optical axis z as shown in Fig. 1. This system images a point source that is situated in the object space at $z = -\infty$ and radiates a linearly polarized monochromatic and coherent electromagnetic wave. This wave is incident upon a lens of aperture Σ that produces a convergent spherical wave in the image space. The origin O of the (x, y, z) coordinate system is positioned at the Gaussian focus. The electric and magnetic fields are determined at the arbitrary point P in the focal region. The aperture size and the distance of P from the aperture was taken to be large compared with the wavelength. In Fig. 1 and what follows, $\hat{s} = (s_x, s_y, s_z)$ is the unit vector along a typical ray in the image space and $\mathbf{r}_p = (x, y, z)$ is the position vector pointing from O to P.

Let $\tilde{\mathbf{E}}(P, t)$ indicate the time-dependent electric field and $\tilde{\mathbf{H}}(P, t)$ indicate the time-dependent magnetic field at P at time t, so that

$$\tilde{\mathbf{E}}(P, t) = \mathrm{Re}[\mathbf{E}(P)\exp(-i\omega t)],$$
$$\tilde{\mathbf{H}}(P, t) = \mathrm{Re}[\mathbf{H}(P)\exp(-i\omega t)], \quad (1)$$

where Re indicates the real part.

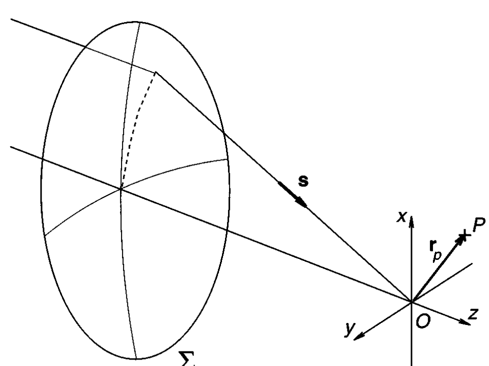

Fig. 1. Diagram showing light focused by a lens into a single medium.

In a homogeneous image space the time-independent electric and magnetic fields can be represented as a superposition of plane waves,[2] and we use the form developed by Wolf[1]:

$$\mathbf{E}(P) = -\frac{ik}{2\pi}\iint_{\Omega}\frac{\mathbf{a}(s_x, s_y)}{s_z}\exp\{ik[\Phi(s_x, s_y) + \hat{s}\cdot\mathbf{r}_p]\} \times \mathrm{d}s_x\mathrm{d}s_y\,, \quad (2)$$

$$\mathbf{H}(P) = -\frac{ik}{2\pi}\iint_{\Omega}\frac{\mathbf{b}(s_x, s_y)}{s_z}\exp\{ik[\Phi(s_x, s_y) + \hat{s}\cdot\mathbf{r}_p]\} \times \mathrm{d}s_x\mathrm{d}s_y\,, \quad (3)$$

where $\Phi(s_x, s_y)$ is the aberration function (describing the optical path difference between the aberrated and the spherical wave front along $\hat{s}$), $\mathbf{a}$ and $\mathbf{b}$ are the electric and magnetic strength vectors, respectively, of the unperturbed electric and magnetic fields in the exit aperture Σ, k is the wave number, and Ω is the solid angle formed by all the geometrical optics rays (and which is therefore a limit for all the $\hat{s}$ unit ray vectors).

Before we proceed with the formulation, it is worth mentioning that Eqs. (2) and (3) represent a summation of the plane waves (unlike the Fresnel–Kirchhoff integral) that are leaving the aperture. It is also evident that the electric and magnetic fields, as represented by Eqs. (2) and (3), do not depend on the particular wave front within the solid angle Ω over which the integration is performed. This statement can be proved in a rigorous mathematical manner.[4] Equations (2) and (3) also show that the phase factor (apart from the aberration function) is the scalar product of the vectors $\hat{s}$ and $\mathbf{r}_p$. It follows from the above that the phase factor expresses the optical path difference between wave fronts going through the point P and the Gaussian focus O, unlike the Fresnel–Kirchhoff integral, for which the phase factor is proportional to the full optical path from the aperture to P.

In this section henceforth we shall not present the derivation of the formulas corresponding to the magnetic field because, apart from the strength vectors, Eqs. (2) and (3) are in the same form.

The case corresponding to Fig. 1, but for an image space consisting of materials 1 and 2 with refractive indices n_1 and n_2, respectively, separated by a planar interface perpendicular to the optical axis, is shown in Fig. 2. The origin O is again positioned at the Gaussian focus. We reformulate Eq. (2) as follows. In material 1 and at the interface ($z = -d$) the incident electric field is given by

$$\mathbf{E}_1(x, y, -d) = -\frac{ik_1}{2\pi}\iint_{\Omega_1}\mathbf{W}^{(e)}(\hat{s}_1)\exp[ik_1(s_{1x}x + s_{1y}y - s_{1z}d)]\mathrm{d}s_{1x}\mathrm{d}s_{1y}\,, \quad (4)$$

where subscripts 1 and 2 denote values corresponding to regions with materials 1 and 2, respectively, the objective lens is taken to be aberration free [$\Phi(s_{1x}, s_{1y}) = 0$], and

$$\mathbf{W}^{(e)}(\hat{s}_1) = \frac{\mathbf{a}(s_{1x}, s_{1y})}{s_{1z}}\,. \quad (5)$$

To describe the field in the second material, we assume that each plane wave component refracting at the interface obeys Fresnel's refraction law, and the resulting field is constructed as a superposition of refracted plane waves. If the amplitude of the plane waves incident upon the interface is described by $\mathbf{W}^{(e)}$, then the amplitude of the transmitted plane waves in the second material is a linear function of $\mathbf{W}^{(e)}$, i.e.,

$$\mathsf{T}^{(2)}\mathbf{W}^{(e)}\,, \quad (6)$$

where the operator $\mathsf{T}^{(2)}$ is a function of the angle of incidence and n_1 and n_2. The transmitted field in the second material at the close vicinity ($z = -d + \delta$) of the interface is given by

$$\mathbf{E}_2(x, y, -d) = -\frac{ik_1}{2\pi}\iint_{\Omega_1}\mathsf{T}^{(e)}\mathbf{W}^{(e)}(\hat{s}_1)\exp[ik_1(s_{1x}x + s_{1y}y - s_{1z}d)]\mathrm{d}s_{1x}\mathrm{d}s_{1y} \quad (7)$$

when $\delta \to 0$. We represent the field inside the second material again as a superposition of plane waves. This representation is a solution of the time-independent wave equation and can be written as

$$\mathbf{E}_2(\mathbf{r}_p) = -\frac{ik_2}{2\pi}\iint_{\Omega_2}\mathbf{F}^{(e)}(\hat{s}_2)\exp(ik_2\hat{s}_2\cdot\mathbf{r}_p)\mathrm{d}s_{2x}\mathrm{d}s_{2y}\,. \quad (8)$$

We must determine the function $\mathbf{F}^{(e)}(\hat{s}_2)$, and for this we shall make use of Eq. (7), which represents a boundary condition for Eq. (8). Furthermore, we shall establish the relationship between $\hat{s}_1$ and $\hat{s}_2$.

It is evident from the vectorial law of refraction,

$$k_2\hat{s}_2 - k_1\hat{s}_1 = (k_2\cos\phi_2 - k_1\cos\phi_1)\mathbf{u}\,, \quad (9)$$

where $\mathbf{u}$ represents the normal of the interface, $\phi_1 = (\hat{s}_1, \mathbf{u})$, and $\phi_2 = (\hat{s}_2, \mathbf{u})$, that

$$k_2 s_{2x} = k_1 s_{1x}, \qquad k_2 s_{2y} = k_1 s_{1y}\,. \quad (10)$$

In what follows, we shall give a solution for Eq. (8) with the boundary condition [Eq. (7)].

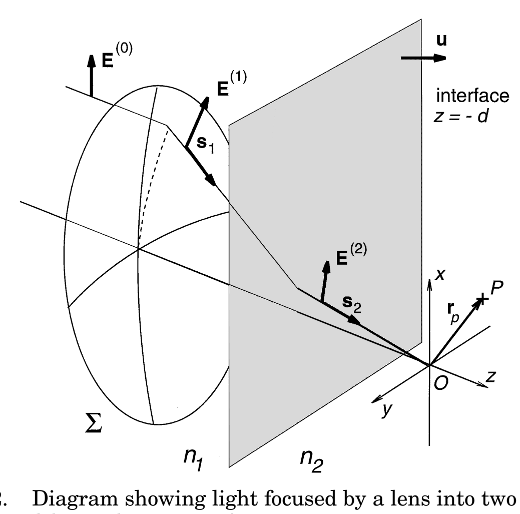

Fig. 2. Diagram showing light focused by a lens into two media separated by a planar interface.

As a result of a coordinate transformation and setting $\hat{s}_2 = f(\hat{s}_1)$, Eq. (8) yields

$$\mathbf{E}_2(\mathbf{r}_p) = -\frac{ik_2}{2\pi}\iint_{\Omega_1}\mathbf{F}^{(e)}(\hat{s}_2)\exp(ik_2\hat{s}_2\cdot\mathbf{r}_p) \times \mathbf{J}_0(s_{1x}, s_{1y}; s_{2x}, s_{2y})\mathrm{d}s_{1x}\mathrm{d}s_{1y}, \quad (11)$$

where $\mathbf{J}_0$ is the Jacobian of the coordinate transformation:

$$\mathbf{J}_0 = \left(\frac{k_1}{k_2}\right)^2,$$

obtained by the use of Eqs. (10). Equation (11) satisfies the boundary condition represented by Eq. (7) when

$$\mathbf{F}^{(e)}(\hat{s}_1, \hat{s}_2) = \left(\frac{k_2}{k_1}\right)\mathsf{T}^{(e)}\frac{\mathbf{a}(s_{1x}, s_{1y})}{s_{1z}}\exp[-id(k_1s_{1z} - k_2s_{2z})]. \quad (12)$$

On substituting from Eq. (12) into Eq. (11), we obtain the electric field in the second material:

$$\mathbf{E}_2(x, y, z) = -\frac{ik_2^2}{2\pi k_1}\iint_{\Omega_1}\mathsf{T}^{(e)}\frac{\mathbf{a}(s_{1x}, s_{1y})}{s_{1z}} \times \exp[-id(k_1s_{1z} - k_2s_{2z})]\exp(ik_2s_{2z}z) \times \exp[ik_1(s_{1x}x + s_{1y}y)]\mathrm{d}s_{1x}\mathrm{d}s_{1y}. \quad (13)$$

The same procedure yields a similar expression for the magnetic vector:

$$\mathbf{H}_2(x, y, z) = -\frac{ik_2^2}{2\pi k_1}\iint_{\Omega_1}\mathsf{T}^{(m)}\frac{\mathbf{b}(s_{1x}, s_{1y})}{s_{1z}} \times \exp[-id(k_1s_{1z} - k_2s_{2z})]\exp(ik_2s_{2z}z) \times \exp[ik_1(s_{1x}x + s_{1y}y)]\mathrm{d}s_{1x}\mathrm{d}s_{1y}. \quad (14)$$

From the above it also follows that in Eqs. (13) and (14) we have

$$s_{2z} = \left(1 - \frac{n_1^2}{n_2^2} + \frac{n_1^2}{n_2^2}s_{1z}^2\right)^{1/2} = \left[1 - \frac{n_1^2}{n_2^2}(s_{1x}^2 + s_{1y}^2)\right]^{1/2}. \quad (15)$$

It is important to emphasize that, since both the boundary condition represented by Eq. (7) and the integral representation [Eq. (8)] are exact solutions of the homogeneous wave equation, our formulas for the electric vector [Eq. (13)] and the magnetic vector [Eq. (14)] in the second material also satisfy the homogeneous wave equation. Therefore we have successfully obtained a consistent extension of Wolf's solution in the second material. We shall show below in Subsection 2.E that the paraxial approximation of Eq. (13) yields the well-known axial distribution[13] for one material, and it also correctly predicts the axial position of the Gaussian focus for two materials.

B. Electric and Magnetic Strength Vectors

The determination of the electric and magnetic strength vectors for a single medium of propagation was described previously.[14] Now we obtain these vectors for a plane polarized wave incident upon the lens, and a material whose refractive index is different from that of the medium of propagation is placed into the image space. The material is taken to be isotropic and homogeneous and to have an optically smooth planar surface that is perpendicular to the optical axis. For our decomposition the usual assumptions are made, namely, that the electric vector maintains its direction with respect to a meridional plane and the electric vector remains on the same side of a meridional plane on passing through the system.

For the optical system under consideration we denote the angle of incidence on the interface by ϕ_1 and the angle of refraction by ϕ_2. The unit vectors $\hat{s}_1$ and $\hat{s}_2$ and the vector $\mathbf{r}_p$ (Fig. 2) are given in spherical polar coordinates by

$$\hat{s}_1 = (\sin\phi_1)(\cos\theta)\hat{\imath} + (\sin\phi_1)(\sin\theta)\hat{\jmath} + (\cos\phi_1)\hat{k}, \quad (16)$$

$$\hat{s}_2 = (\sin\phi_2)(\cos\theta)\hat{\imath} + (\sin\phi_2)(\sin\theta)\hat{\jmath} + (\cos\phi_2)\hat{k}, \quad (17)$$

$$\mathbf{r}_p = r_p[(\sin\phi_p)(\cos\theta_p)\hat{\imath} + (\sin\phi_p)(\sin\theta_p)\hat{\jmath} + (\cos\phi_p)\hat{k}], \quad (18)$$

where $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are the unit base vectors of the (x, y, z) orthogonal system and the spherical polar coordinates r, ϕ, and θ are defined so that $r > 0$, $0 \le \phi < \pi$, and $0 \le \theta < 2\pi$. The coordinate system is chosen so that the y component of the incident electric vector is zero. For the incident electric vector $\mathbf{E}^{(0)}$ in front of the lens we have

$$\mathbf{E}^{(0)} = \begin{pmatrix} E_0 \\ 0 \\ 0 \end{pmatrix}.$$

For a treatment of the refraction that occurs at the interface of the mismatched materials it is convenient to decompose the electric vector into s- and p-polarized vector components E_s and E_p, respectively, and to rotate the coordinate system so that the new coordinate system will contain components in the (p, s, ζ) system. This coordinate system is defined in such a way that $E_\zeta = 0$. The electric vector components $\mathbf{E}^{(1)}_{(p,s,\zeta)}$ after the lens are then in the form

$$\mathbf{E}^{(1)}_{(p,s,\zeta)} = A(\phi_1)\mathbf{P}^{(1)}\mathbf{L}\,\mathbf{R}\,\mathbf{E}^{(0)}_{(x,y,z)}, \quad (19)$$

where $A(\phi_1)$ is an amplitude function (defined below), the matrix $\mathbf{R}$ describes the coordinate transformation for rotation around the z axis,

$$\mathbf{R} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (20)$$

the matrix $\mathbf{L}$ describes the changes in the electric field as it traverses the lens,

$$\mathbf{L} = \begin{bmatrix} \cos\phi_1 & 0 & \sin\phi_1 \\ 0 & 1 & 0 \\ -\sin\phi_1 & 0 & \cos\phi_1 \end{bmatrix}, \quad (21)$$

and the matrix $\mathbf{P}^{(1)}$ describes the coordinate system rotation that generates E_s and E_p components with $E_\zeta = 0$,

$$\mathbf{P}^{(1)} = \begin{bmatrix} \cos\phi_1 & 0 & -\sin\phi_1 \\ 0 & 1 & 0 \\ \sin\phi_1 & 0 & \cos\phi_1 \end{bmatrix}. \quad (22)$$

It is interesting to note that in our case, when $E_z^{(0)} = 0$, the operations by $\mathbf{L}$ and $\mathbf{P}^{(1)}$ cancel out.

The electric field in the second material, $\mathbf{E}^{(2)}$, is given by

$$\mathbf{E}^{(2)}_{(x,y,z)} = \mathbf{R}^{-1}[\mathbf{P}^{(2)}]^{-1}\mathbf{I}\,\mathbf{E}^{(1)}_{(p,s,\zeta)}\,, \tag{23}$$

where the matrix $\mathbf{I}$ describes the effect of the interface,

$$\mathbf{I} = \begin{bmatrix} \tau_p & 0 & 0 \\ 0 & \tau_s & 0 \\ 0 & 0 & \tau_p \end{bmatrix}, \tag{24}$$

in which τ_p and τ_s are the Fresnel coefficients,

$$\tau_s = \frac{2 \sin \phi_2 \cos \phi_1}{\sin(\phi_1 + \phi_2)}\,, \qquad \tau_p = \frac{2 \sin \phi_2 \cos \phi_1}{\sin(\phi_1 + \phi_2)\cos(\phi_1 - \phi_2)}\,, \tag{25}$$

the matrix $[\mathbf{P}^{(2)}]^{-1}$ describes the rotation of the coordinate system required to return it to the (p, s, z) system,

$$[\mathbf{P}^{(2)}]^{-1} = \begin{bmatrix} \cos \phi_2 & 0 & \sin \phi_2 \\ 0 & 1 & 0 \\ -\sin \phi_2 & 0 & \cos \phi_2 \end{bmatrix}, \tag{26}$$

and the matrix $\mathbf{R}^{-1}$ describes the inverse rotation around the z axis.

From Eqs. (19)–(21) and (23) we obtain the components of the electric vector in the second material on setting $E_0 = 1$:

$$\mathbf{E}^{(2)}_{(x,y.z)} = A(\phi_1)\begin{pmatrix} \tau_p \cos \phi_2 \cos^2 \theta + \tau_s \sin^2 \theta \\ \tau_p \cos \phi_2 \sin \theta \cos \theta - \tau_s \sin \theta \cos \theta \\ -\tau_p \sin \phi_2 \cos \theta \end{pmatrix}. \tag{27}$$

The function $A(\phi_1)$ can be regarded as an apodization function that depends on the lens used in imaging. Richards and Wolf[14] showed that when the system obeys Abbe's sine condition, i.e., is aplanatic, then

$$A(\phi_1) = f l_0 \cos^{1/2} \phi_1\,, \tag{28}$$

where f is the focal length of the lens *in vacuo* and l_0 is an amplitude factor. Otherwise, this function can take different forms.[11]

Having derived the electric vector $\mathbf{E}^{(2)}_{(x,y,z)}$, we can write the electric strength vector $\mathbf{c}$ as

$$\mathbf{c} = \mathsf{T}^{(e)}\mathbf{a} = \mathbf{E}^{(2)}\,. \tag{29}$$

The magnetic strength vector $\mathbf{d}$ is then given by

$$\mathbf{d} = \left(\frac{\epsilon_2}{\mu_2}\right)^{1/2} \hat{s}_2 \times \mathbf{c}\,, \tag{30}$$

where μ_2 is the permeability and ϵ_2 is the permittivity of the second material. The magnetic strength vector from Eqs. (17), (27), (29), and (30) is given by

$$\mathbf{d} = \left(\frac{\epsilon_2}{\mu_2}\right)^{1/2} A(\phi_1) \times \begin{pmatrix} -\tau_p \cos \theta \sin \theta + \tau_s \sin \theta \cos \theta \cos \phi_2 \\ \tau_p \cos^2 \theta + \tau_s \sin^2 \theta \cos \phi_2 \\ -\tau_s \sin \theta \cos \phi_2 \end{pmatrix}. \tag{31}$$

We note that, for the special case $\phi_1 = \phi_2$, $\epsilon_1 = \epsilon_2 = 1$, and $\mu_1 = \mu_2 = 1$ (in the CGS unit system), Eqs. (27) and (31) reduce to the electric strength vector given by Richards and Wolf,[14] as they should.

C. Electric and Magnetic Field Vectors

First we formulate the expressions needed to simplify Eqs. (13) and (14). We transform the integral variables $\mathrm{d}s_{1x}$ and $\mathrm{d}s_{1y}$ to the spherical polar coordinate system. The Jacobian of the transformation is

$$\mathbf{J}_p = \sin \phi_1 \cos \phi_1\,;$$

thus

$$\mathrm{d}s_{1x}\mathrm{d}s_{1y} = \sin \phi_1 \cos \phi_1 \mathrm{d}\phi_1 \mathrm{d}\theta\,. \tag{32}$$

We define

$$\kappa = n_1 \sin \phi_1 \sin \phi_p \cos(\theta - \theta_p) + n_2 \cos \phi_2 \cos \phi_p\,, \tag{33}$$

$$\Psi(\phi_1, \phi_2, -d) = -d(n_1 \cos \phi_1 - n_2 \cos \phi_2)\,. \tag{34}$$

So far we have constructed the quantities that we require to determine the electric and magnetic vectors. With the help of the above results we can rewrite Eqs. (13) and (14) to express the electric ($\mathbf{E}_2$) and magnetic ($\mathbf{H}_2$) vectors inside the second material:

$$\mathbf{E}_2(\mathbf{r}_p, -d) = -\frac{ik_2^2}{2\pi k_1}\iint_{\Omega_1} \mathbf{c}(\phi_1, \phi_2, \theta) \times \exp\{ik_0[r_p\kappa + \Psi(\phi_1, \phi_2, -d)]\} \times \sin \phi_1 \mathrm{d}\phi_1 \mathrm{d}\theta\,, \tag{35}$$

$$\mathbf{H}_2(\mathbf{r}_p, -d) = -\frac{ik_2^2}{2\pi k_1}\iint_{\Omega_1} \mathbf{d}(\phi_1, \phi_2, \theta) \times \exp\{ik_0[r_p\kappa + \Psi(\phi_1, \phi_2, -d)]\} \times \sin \phi_1 \mathrm{d}\phi_1 \mathrm{d}\theta\,, \tag{36}$$

where k_0 indicates the wave number *in vacuo*. On substituting from Eqs. (27), (29), (31), and (32) into Eqs. (35) and (36), changing the integration limits, and assuming that the system obeys the sine condition [Eq. (28)], we obtain the following expressions for the electric-field components:

$$e_{2x} = +\frac{iK}{2\pi}\int_0^{\alpha}\int_0^{2\pi} (\cos \phi_1)^{1/2}(\sin \phi_1) \times [(\tau_p \cos \phi_2 + \tau_s) + (\cos 2\theta)(\tau_p \cos \phi_2 - \tau_s)] \times \exp\{ik_0[r_p\kappa + \Psi(\phi_1, \phi_2, -d)]\}\mathrm{d}\phi_1\mathrm{d}\theta\,,$$

$$e_{2y} = +\frac{iK}{2\pi}\int_0^{\alpha}\int_0^{2\pi} (\cos \phi_1)^{1/2}(\sin \phi_1)(\sin 2\theta) \times (\tau_p \cos \phi_2 - \tau_s) \times \exp\{ik_0[r_p\kappa + \Psi(\phi_1, \phi_2, -d)]\}\mathrm{d}\phi_1\mathrm{d}\theta\,,$$

$$e_{2z} = -\frac{iK}{\pi}\int_0^{\alpha}\int_0^{2\pi} (\cos \phi_1)^{1/2}(\sin \phi_1)\tau_p \sin \phi_2 \cos \theta \times \exp\{ik_0[r_p\kappa + \Psi(\phi_1, \phi_2, -d)]\}\mathrm{d}\phi_1\mathrm{d}\theta\,, \tag{37}$$

where

$$K = \frac{k_2^2 f l_0}{2k_1} = \frac{\pi n_2^2 f l_0}{\lambda n_1}\,.$$

We also obtain the following expressions for the magnetic-field components:

$$h_{2x} = +\frac{iKn_2}{2\pi}\int_0^{\alpha}\int_0^{2\pi}(\cos\phi_1)^{1/2}(\sin\phi_1)(\sin 2\theta)\times(\tau_s\cos\phi_2-\tau_p)\times\exp\{ik_0[r_p\kappa+\Psi(\phi_1,\phi_2,-d)]\}\mathrm{d}\phi_1\mathrm{d}\theta\,,$$
$$h_{2y} = +\frac{iKn_2}{2\pi}\int_0^{\alpha}\int_0^{2\pi}(\cos\phi_1)^{1/2}(\sin\phi_1)\times[(\tau_p+\tau_s\cos\phi_2)+(\tau_p+\tau_s\cos\phi_2)(\cos 2\theta)]\times\exp\{ik_0[r_p\kappa+\Psi(\phi_1,\phi_2,-d)]\}\mathrm{d}\phi_1\mathrm{d}\theta\,,$$
$$h_{2z} = -\frac{iKn_2}{\pi}\int_0^{\alpha}\int_0^{2\pi}(\cos\phi_1)^{1/2}(\sin\phi_1)\tau_s\sin\phi_2\sin\theta\times\exp\{ik_0[r_p\kappa+\Psi(\phi_1,\phi_2,-d)]\}\mathrm{d}\phi_1\mathrm{d}\theta\,, \quad (38)$$

where we assumed that $\mu_2 = 1$ and set α to be the angular semiaperture of the lens. The integration in Eqs. (37) and (38) can be carried out[14] with respect to θ, and from the result the electric- and magnetic-field components can be expressed as the combination of two sets of three integrals, I_0, I_1, and I_2:

$$e_{2x} = -iK[I_0^{(e)} + I_2^{(e)}\cos(2\theta_p)]\,,$$
$$e_{2y} = -iKI_2^{(e)}\sin(2\theta_p)\,,$$
$$e_{2z} = -2KI_1^{(e)}\cos\theta_p\,, \quad (39)$$
$$h_{2x} = -iKn_2I_2^{(h)}\sin(2\theta_p)\,,$$
$$h_{2y} = -iKn_2[I_0^{(h)} - I_2^{(h)}\cos(2\theta_p)]\,,$$
$$h_{2z} = -2Kn_2I_1^{(h)}\sin\theta_p\,, \quad (40)$$

where θ_p is defined in Eq. (18) and we have put $n_2 = \sqrt{\epsilon_2}$. After we substitute the normalized optical coordinates

$$v = k_1(x^2+y^2)^{1/2}\sin\alpha = k_1r_p\sin\phi_p\sin\alpha\,,$$
$$u = k_2z\sin^2\alpha = k_2r_p\cos\phi_p\sin^2\alpha \quad (41)$$

into Eqs. (37) and (38), the integrals I_0, I_1, and I_2 are given by

$$I_0^{(e)} = \int_0^{\alpha}(\cos\phi_1)^{1/2}(\sin\phi_1)\exp[ik_0\Psi(\phi_1,\phi_2,-d)]\times(\tau_s+\tau_p\cos\phi_2)J_0\left(\frac{v\sin\phi_1}{\sin\alpha}\right)\times\exp\left(\frac{iu\cos\phi_2}{\sin^2\alpha}\right)\mathrm{d}\phi_1\,,$$
$$I_1^{(e)} = \int_0^{\alpha}(\cos\phi_1)^{1/2}(\sin\phi_1)\exp[ik_0\Psi(\phi_1,\phi_2,-d)]\times\tau_p(\sin\phi_2)J_1\left(\frac{v\sin\phi_1}{\sin\alpha}\right)\exp\left(\frac{iu\cos\phi_2}{\sin^2\alpha}\right)\mathrm{d}\phi_1\,,$$
$$I_2^{(e)} = \int_0^{\alpha}(\cos\phi_1)^{1/2}(\sin\phi_1)\exp[ik_0\Psi(\phi_1,\phi_2,-d)]\times(\tau_s-\tau_p\cos\phi_2)J_2\left(\frac{v\sin\phi_1}{\sin\alpha}\right)\times\exp\left(\frac{iu\cos\phi_2}{\sin^2\alpha}\right)\mathrm{d}\phi_1\,; \quad (42)$$

$$I_0^{(h)} = \int_0^{\alpha}(\cos\phi_1)^{1/2}(\sin\phi_1)\exp[ik_0\Psi(\phi_1,\phi_2,-d)]\times(\tau_p+\tau_s\cos\phi_2)J_0\left(\frac{v\sin\phi_1}{\sin\alpha}\right)\times\exp\left(\frac{iu\cos\phi_2}{\sin^2\alpha}\right)\mathrm{d}\phi_1\,,$$
$$I_1^{(h)} = \int_0^{\alpha}(\cos\phi_1)^{1/2}(\sin\phi_1)\exp[ik_0\Psi(\phi_1,\phi_2,-d)]\times\tau_s(\sin\phi_2)J_1\left(\frac{v\sin\phi_1}{\sin\alpha}\right)\exp\left(\frac{iu\cos\phi_2}{\sin^2\alpha}\right)\mathrm{d}\phi_1\,,$$
$$I_2^{(h)} = \int_0^{\alpha}(\cos\phi_1)^{1/2}(\sin\phi_1)\exp[ik_0\Psi(\phi_1,\phi_2,-d)]\times(\tau_p-\tau_s\cos\phi_2)J_2\left(\frac{v\sin\phi_1}{\sin\alpha}\right)\times\exp\left(\frac{iu\cos\phi_2}{\sin^2\alpha}\right)\mathrm{d}\phi_1\,, \quad (43)$$

where J_n is the Bessel function of the first kind, of order n. Equations (39), (40), (42), and (43) conclude our solution of the problem. It should be noted that, for $n_1 = n_2 = 1$ (therefore $\phi_2 = \phi_1$ and $d = 0$), Eqs. (42) and (43) reduce to $I_n^{(e)} = I_n^{(h)} = I_n$, which are then identical to the corresponding equations of Richards and Wolf.[14] Therefore our formulation of the vector diffraction theory with spherical aberration introduced by an interface between two materials of mismatched refractive indices is consistent with previous results derived for the nonaberrated case.

We can see from Eqs. (42) and (43) that the time-independent electric and magnetic fields can be described not by only one set of integral functions but rather by two such sets. The reason is that the electric and magnetic vectors are orthogonal with respect to each other and the polarization-dependent refraction at the interface acts differently on different electric and magnetic vector directions. It is emphasized, however, that all the laws that describe the transition of these two vectors through the system are essentially the same, and the difference between the two sets of integrals does not originate from the nature of the wave (whether it is an electric or a magnetic wave) but rather depends on whether the wave incident upon the interface possesses p or s polarization.

D. Physical Interpretation

Although the formal solution of our problem has been concluded by Eqs. (39) and (40), we recall here Eq. (13) for an analysis of our results. This latter equation can be rewritten in the form

$$\mathbf{E}_2(x, y, z) = K'\iint_{\Omega_1}\mathbf{W}_0(s_{1x}', s_{1y}')\times\exp[i2\pi(s_{1x}'x, s_{1y}'y)]\mathrm{d}s_{1x}'\mathrm{d}s_{1y}'\,, \quad (44)$$

where

$$s_{1x}' = \frac{n_1s_{1x}}{\lambda}\,, \qquad s_{1y}' = \frac{n_1s_{1y}}{\lambda}\,, \quad (45)$$

$$\mathbf{W}_0 = \frac{\mathbf{c}(s_{1x}, s_{1y})}{s_{1z}}\exp[-i(k_1s_{1z}-k_2s_{2z})d]\exp(ik_2s_{2z}z) = \frac{\mathbf{c}(s_{1x}, s_{1y})}{s_{1z}}\exp(-ik_1s_{1z}d)\exp[ik_2s_{2z}(z+d)]\,, \quad (46)$$

and K' is constant. If we extend the limit of integration in Eq. (44) to the range $[-\infty, \infty]$, the latter equation becomes a Fourier integral. The physical meaning of the above extension is that the integration is performed over the full π solid angle and not only over the area of the aperture. In other words, the integral operation is being performed not only in the range

$$\left(\frac{k_1}{k_2}\right)^2 ({s_{1x}}^2 + {s_{1y}}^2) < 1 \tag{47}$$

but also outside this region. Relation (47) represents the condition for homogeneous waves. Thus, when the integration limits are extended to $[-\infty, \infty]$, the integral representation also includes evanescent waves into the resulting field. These evanescent waves propagate perpendicular to the optical axis, i.e., very close and parallel to the plane of the aperture. These waves decay exponentially at larger z distances from the aperture and therefore do not contribute to the far-field distribution.

We can conclude that the electromagnetic field of the optical system under consideration can be represented in the second material as the Fourier transform of a function $\mathbf{W}_0$, which consists of an amplitude factor and two phase factors as shown by Eq. (46). These two phase factors represent the phase at the point of observation (P) with respect to the Gaussian focus (O); i.e., one of these phase factors represents the phase $\exp[ik_2 s_{2z}(z + d)]$ as the wave propagates in the second material, and the other one represents the phase $\exp(-ik_1 s_{1z} d)$ that the wave would have had in the absence of the second material. The sign of the latter phase component is negative, and so the total of the two phases gives the phase difference with respect to the Gaussian focus.

There is an alternative way to write Eq. (44):

$$\mathbf{E}_2(x, y, z) = K' \iint_{\Omega_1} \mathbf{W}_0'(s_{1x}', s_{1y}')\exp(ik_2 s_{2z} z) \times \exp[i2\pi(s_{1x}'x + s_{1y}'y)]\mathrm{d}s_{1x}'\mathrm{d}s_{1y}', \tag{48}$$

where

$$\mathbf{W}_0' = \frac{\mathbf{W}_0}{\exp(ik_2 s_{2z} z)} \, .$$

Equation (48) is formally identical to the angular spectrum representation given by Goodman.[15] Thus we can conclude that our extension of the Debye integral for the optical system under consideration can also be regarded as the angular spectrum representation. The spectrum is given as an amplitude factor describing the electric (and also the magnetic) field as it traverses the system and a phase factor corresponding to the phase difference between the ray vectors along the z direction (which latter phase factor, apart from the distance d, is often referred to as the astigmatic constant).

It should also be mentioned that Eq. (35) is in a form identical to that of the corresponding equation of Wolf,[1] i.e., the aberration function is represented in an *exact* form by $\Psi(\phi_1, -d)$, as defined by Eq. (34). Since this function is dependent only on the azimuthal angle ϕ_1, it describes spherical aberration.

E. Paraxial Approximation

Equation (13) can be approximated paraxially so as to correspond to the axial light distribution for a low-aperture objective lens. For this we recall the expression for the electric field:

$$\mathbf{E}_2(x, y, z) = -\frac{ik_2^2}{2\pi k_1} \iint_{\Omega_1} \mathsf{T}^{(e)} \frac{\mathbf{a}(s_{1x}, s_{1y})}{s_{1z}} \times \exp[-id(k_1 s_{1z} - k_2 s_{2z})] \times \exp(ik_2 s_{2z} z)\exp[ik_1(s_{1x}x + s_{1y}y)]\mathrm{d}s_{1x}\mathrm{d}s_{1y} \, . \tag{49}$$

In the paraxial approximation the strength vector $\mathsf{T}^{(e)}\mathbf{a}$ can be considered as constant, and therefore it is independent of the integration. Furthermore, we can set $1/s_{1z}$ = constant and $x = y = 0$. The z components of the unit ray vectors can be approximated with the first and second terms of their expansion:

$$s_{1z} = [1 - ({s_{1x}}^2 + {s_{1y}}^2)]^{1/2} \approx 1 - \frac{{s_{1x}}^2 + {s_{1y}}^2}{2} = 1 - \frac{\sigma^2}{2},$$

$$s_{2z} = [1 - ({s_{2x}}^2 + {s_{2y}}^2)]^{1/2} \approx 1 - \frac{{s_{2x}}^2 + {s_{2y}}^2}{2} = 1 - \left(\frac{n_1}{n_2}\right)^2 \frac{\sigma^2}{2} \, . \tag{50}$$

After we rearrange Eq. (49) and perform further calculations, the paraxial approximation of the electric field, $E_2^{(\mathrm{par})}$, is derived as

$$E_2^{(\mathrm{par})}(0, 0, z) = -\frac{ik_2^2}{k_1} B \exp\left\{-k_1\left[d\left(1 - \frac{n_2}{n_1}\right) + \frac{n_2}{n_1} z\right]\right\} \times \int_0^\beta \exp\left(-\frac{1}{2} ik_1 D {\sigma_1}^2\right)\sigma\mathrm{d}\sigma \, , \tag{51}$$

where β is the paraxial semiangle of the aperture, B is a constant, and

$$D = \left(1 - \frac{n_1}{n_2}\right)d + \frac{n_1}{n_2} z \, . \tag{52}$$

After we perform the integration with respect to σ, the intensity is given by

$$I(0, 0, z) = |E_2^{(\mathrm{par})}(0, 0, z)|^2 = I_0 \left| \frac{\exp\left(-\frac{1}{2} ik_1 D\beta^2\right) - 1}{\frac{1}{4} k_1 D \beta^2} \right|^2 = 4I_0 \frac{\sin^2\left(\frac{1}{4} k_1 D\beta^2\right)}{\left(\frac{1}{4} k_1 D\beta^2\right)^2} \, . \tag{53}$$

By putting $n_1 = n_2$ in Eqs. (52) and (53), we obtain the well-known axial intensity distribution for a single material. If we use the integral formalism $I_1^{(e)} = I_2^{(e)} = 0$ in the paraxial approximation, $I_0^{(e)}$ yields a form identical to that of Eq. (51). By putting $D = 0$ in Eq. (52), we obtain the location of the paraxial focus, for the case of two materials, that corresponds to that obtained by geometrical optics.

In a subsequent paper we will show how our integral formulas can be solved analytically, and we will present results for the numerically computed time-averaged electric energy density as a function of the numerical aperture of the lens and the depth of focusing into the bulk specimen.

It is emphasized that our solution for the extension of the Debye integral is an exact solution of the homogeneous wave equation and is therefore valid everywhere except in the immediate vicinity of the aperture.

3. CONCLUSIONS

We have presented a new solution of electromagnetic diffraction for the problem of light focused into an isotropic and homogeneous material with refractive index different from that of the medium of propagation and situated in the image space of a high aperture coherent optical system. Focusing into such materials results in spherical aberration.

We have solved the above diffraction problem for the case of a planar interface in a rigorous mathematical manner, and the solution satisfies the homogeneous wave equation and is therefore valid everywhere except in the immediate vicinity of the aperture. The solution can be regarded as an extension of Wolf's integral formulas, for focusing in a single medium, and is valid for high aperture focusing into materials of mismatched refractive indices. We have shown that for our case the aberration function is in a closed analytical form. We have obtained the electric and magnetic strength vectors in the second material. We have given a physical interpretation of our results and have obtained the paraxial approximation of our solution.

The main advantages of our method are as follows. The solution is in a simple form that can be directly used for numerical computation. The method for obtaining the strength vectors is generally applicable. Incident electric vectors, with directions other than perpendicular to the optical axis, can readily be treated. The solution is consistent, and, for the special case of a single medium of propagation, it reduces to the results published previously. Apart from the initial assumption of Wolf's integral formulas, no approximations are used.

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